

to God. "Instead of a God for whom all is already made, to whom all is given, we have a God who acts freely in an open universe" (p. 189).

Immortality is neither denied nor affirmed by this philosophy, though it denies a timeless and unchanging soul. On the other hand it gives in the doctrine of pure duration a hope that individual histories may be somehow preserved.

The Idea of a Reality Which Creates and Is Free (chap. ix).—Free action is creation, and is the opposite of mechanical repetition. Two things distinguish vital and conscious action from action which is mechanical and unconscious. (1) The conditions of a conscious action cannot be repeated. (2) In conscious action there are new determinants of a non-mechanical kind, to wit, purposes. It is vain to look for freedom in science. The intellectual view is unavoidably rigid and deterministic. But there remains intuition. The free act is the manifestation of the whole personality. But of freedom there is nevertheless a condition—"this condition is an open universe". The notion of an Absolute which is perfect and complete is compared with that of an Absolute which is a life that endures.

"This metaphysical conception of life as the reality which creates and is free is actually moulded on experience. The philosophy of change is not therefore a logic-tight system, complete and perfect, from which we can take nothing and to which we can add nothing. It has nothing systematic about it. It has not an answer for every question. It is a method which distinguishes different problems and examines them separately. Philosophy, like physical science, is capable of infinite progress to ever greater perfection" (p. 213).

Such in bare outline, is the argument of *The Philosophy of Change*; I can only hope that my own private difficulties have not altogether prevented me from doing it justice. For I must needs suspect the adequacy of my exposition, since I am unable to accept Bergson's view of the intelligence, and so, lacking the initial vision, I no doubt fail to understand. And further it seems to me that the "unconscious" wilts visibly under the strain put upon it in the theory of memory. All the same, after reading nearly all the works put forth on the new philosophy for several years, I am sure that Mr. Carr's book stands almost, perhaps quite, alone in interest, lucidity and importance.

ARTHUR ROBINSON.

A Theory of Time and Space. By ALFRED A. ROBB. Cambridge University Press. Pp. vi, 373.

EVERYONE who read two small pamphlets by Mr. Robb, one called *Optical Geometry of Motion*, and the other with the same

title as the present work, will be greatly pleased that the more elaborate treatment foreshadowed in them has now been completed and published. The pamphlets were reviewed in *MIND* by the present writer recently. Mr. Robb's new book is most important and interesting, but it is not easy to review in a non-technical way. After a short philosophical preface, Mr. Robb introduces us to the notion of 'Conical Order'; this part of the book is practically a reproduction of his second pamphlet. He then lays down a number of postulates about *before* and *after* such that the elements in the field of these relations shall stand in a conical order. From these postulates he deduces two hundred and six theorems. The upshot of the matter is that the field of *before* and *after* is shown to be a manifold in which any element can be represented by four coordinates; three of these have the properties that we commonly ascribe to spatial coordinates, the fourth has those that we commonly ascribe to time. But, since the elements of which this geometry is composed are simply defined as constituting the field of *before* and *after*, and the postulates defining *before* and *after* are themselves obtained by considering the temporal relations of events, Mr. Robb concludes that he has succeeded in defining space in terms of time.

No philosopher interested in the foundations of physics can afford to neglect Mr. Robb's contentions. I think I shall best make Mr. Robb's position clear to the intending reader if I discuss shortly the following points: (1) The meaning of Conical Order and the reasons for supposing that instants stand in a conical order; (2) Some of the special notions introduced and defined by Mr. Robb, and their relations to the geometry of the cone; (3) The real philosophical meaning and importance of work on Mr. Robb's lines. I shall assume, what I have seen no reason to doubt, that the theorems really do follow from the postulates. I may remark in passing that those who are interested in symbolic logic will find it a very good exercise to state the postulates of the book formally and then to prove some of the more important theorems for themselves by the methods of *Principia Mathematica*.

(1) A relation is said to generate a conical order when it is transitive and aliorrelative but not connexive, and fulfils certain other conditions. A very simple example of a relation that fulfils the first three is the relation *north of*. It is transitive; for the fact that Cambridge is north of London and York north of Cambridge implies that York is north of London. It is aliorrelative; for no place is north of itself. It is not connexive; for two places, though each north of other places, may be neither north nor south of each other, since both may lie on the same parallel of latitude. Such relations are not serial, but it may be possible to find classes of terms in their fields which shall be in serial relations. *E.g.* the places on any one meridian of longitude are in a series. Mr. Robb calls the order generated by certain relations *conical* for the following

reason. Suppose we take any definite direction in ordinary space, and make every point in space the vertex of a cone with a fixed vertical angle and an axis parallel to this direction. Let us call α the relation that any point within the upper part of one of these cones has to the vertex of the cone. Any point in the lower part of one of these cones will have the converse relation $\bar{\alpha}$ to the vertex of the cone. Then α is a relation which is transitive, aliorelative, and non-connexive. The first two properties obviously belong to α ; the last can be seen to belong if we notice that there are many points which are neither in the upper nor the lower cones through a given point. All such points have neither the relation α nor $\bar{\alpha}$ to the given point. The surfaces of the cones through any point P are called respectively the α - and β - subsets of P. (Mr. Robb uses β to stand for $\bar{\alpha}$.) We must notice that the cones are only used as illustrations, and that they only provide a satisfactory illustration for a three-dimensional manifold of elements. Mr. Robb's manifold is four-dimensional, but this does not prevent him from defining a conical order and α - and β - subsets in such a way as to agree with the geometrical illustration when we imagine the number of dimensions reduced to three.

So far we have merely been dealing with the logical properties of certain relations of which the relation α is an illustration. Now we come to a question partly of fact and partly of convention. Mr. Robb assumes that the relation of *before* and *after* between events is conical and not serial as has generally been supposed. This means that he assumes that of two events one may be neither before nor after the other, and yet may not be simultaneous with the other. Why should he assume this, which seems so paradoxical at first sight? His argument comes to this: I have two different means of judging about the temporal relations of events. If I directly experience the events I can make direct judgments about their temporal relations. If I do not directly experience both the events, but believe that one causes the other, I can be sure that the cause must proceed the effect. This Mr. Robb takes as an axiom.

Suppose that at a moment t_1^a I send out a flash of light from A to a mirror at B. Let it reach B at t_2^b and be immediately reflected back to A, reaching me there at t_3^a . Then the axiom tells me that t_2^b is after t_1^a and before t_3^a . And direct experience tells me that t_3^a is after t_1^a . But it seems that no influence travels faster than light. Hence no influence leaving B at t_2^b will reach A before t_2^a . We have therefore no reason to suppose that t_2^b is before any moment at A which is before t_2^a . Similarly no influence that leaves A after t_1^a can reach B at t_2^b . We have therefore no reason to suppose that t_2^b is after any moment that is after t_1^a . We have therefore no reason to suppose that t_2^b is either before or after any moment at A that is between t_1^a and t_2^a . And neither of our criteria enables us to judge that t_2^b is simultaneous with *any*

moment at A between t_1^a and t_2^a , still less to decide *which* particular moment it is simultaneous with. It is as a rule tacitly assumed that $t_2^b = \frac{1}{2}(t_1^a + t_2^a)$. Mr. Robb rejects this suggestion because, as we know, it leads when combined with the *facts* (as distinct from any theory) of relativity to the paradoxical results that events, simultaneous when observed from one system, are not so when observed from another. He holds that any assumption that leads to such a result must be rejected, because it is a fundamental law of logic that 'a thing cannot both be and not be at the same time'. I am not quite clear how Mr. Robb means to apply this principle to the present case. If he means that on the ordinary theory two events e_1 and e_2 are both simultaneous and not simultaneous at the same time, because in S_1 they both occur at t while in S_2 the one nearer the origin occurs later than the one further off, I should suppose that the answer is that there is no logical difficulty, because no one supposes that their simultaneity and non-simultaneity subsist at the *same* time. This would be inconsistent with the Theory of Relativity which refuses to recognise a time common to both S and S_2 , but simply holds that the laws of physics can be stated equally well in terms of the local time of *any* non-accelerated system. If, on the other hand, Mr. Robb means that if logic is to apply to all systems there must be a time common to all systems, I do not agree. We should only get into logical difficulties if from any system S' we were forced to judge that incompatible events occurred simultaneously in a system S . But incompatibility in physical matters is always spatio-temporal; e.g. we should need to judge that there was a green and a red flash at the same time and *in the same place* before we should find any logical difficulty. Now the ordinary theory of relativity never forces us to do this. It is only simultaneous events which occur at *different* places in one system that can be judged to be successive from another, and it is only successive events that occur at *different* places in one system that can be judged to be simultaneous from another.

However this may be, Mr. Robb prefers to assume that the relation of *before* and *after* between moments really is non-connective, i.e. that certain moments are neither before nor after each other and yet not identical, and that this is not merely a matter of our inability to find a satisfactory test for their temporal relations in certain cases.

Before leaving this part of the subject I have two criticisms to make. (a) One would like to know how Mr. Robb is defining cause and effect. If he is merely defining it as it is commonly defined in physics as functional correlation, I fail to see why cause must proceed effect, or what precisely this means. If he is using it in some other sense we should wish to know what is the characteristic that distinguishes a cause from an effect. It must of course be an observable one, or the criterion will be useless. (b) Mr. Robb in this introduction does not make quite clear what he con-

siders to be the relation between (i.) the linear set of events in a single experience; (ii.) the linear set of events that happen to a single material particle; and (iii.) the statement that the only simultaneous events are those that happen in the same place. Are simultaneous psychical events in my mind in the same place; and, if so, in what place? Hardly in that of any *one* material particle in my brain; but, if in several, then simultaneous events do happen at different places.

(2) A good many of Mr. Robb's special notions can be easily illustrated from the geometry of the cone, though we must always remember that this forms an incomplete illustration, because the manifold of moments is for him four-dimensional. Thus an *Optical Line* is represented by a generator of a standard cone; an *Inertia Line* is represented by a straight line through the vertex that falls within a standard cone; and a *Separation Line* is represented by a straight line through the vertex that lies outside a standard cone. If we regard the axis of the cone as representing time elapsed (using time in the ordinary sense), and the three other coordinates as representing space passed over in the ordinary sense, we can see that an optical line represents the successive positions of an element of a wave-front sent out from the vertex at time 0 and travelling in vacuo, provided that the vertical angle of the standard cone is $\tan^{-1} c$ where c is the velocity of light. An inertia line represents the motion of any actual unaccelerated particle, assuming that nothing can travel faster than light. A separation line cannot represent the motion of any particle for this would mean that the particle travelled faster than light; any two points on it must therefore represent separate and distinct particles. Similarly we get three kinds of planes—optical planes, acceleration planes and separation planes. The conical analogies are respectively tangent planes, planes that cut the cone in two real lines, and planes that cut it in two imaginary lines. We also get three kinds of parallelism among optical lines.

As our manifold of instants is four-dimensional we shall have to consider *threefolds* as well as lines and planes. Here of course we cannot offer any geometrical illustration that shall be wholly satisfactory. A general threefold is the aggregate of all elements in any general plane P which intersects any general line a and of all the elements in all planes parallel to P that intersect a . (A *general* line means simply a line which is either optical, inertia, or separation, and a general plane means one which is either optical, acceleration, or separation. It is proved that these alternatives are exhaustive and exclusive, as can be seen from the geometrical illustrations taken from the cone.) It is found that there are three distinct kinds of threefold: these are called optical, separation, and rotation threefolds according as the general line a is an optical, separation, or inertia line.

Mr. Robb proves the extremely interesting and important result

that the geometry of a separation-threefold with his postulates is Euclidean. Before this he has had of course to introduce the notion of congruence. Congruence has to be defined differently for the different types of line, and segments on different types of line are not congruent with each other. Now the only kind of threefold that contains only lines and planes of a single type (*viz.* separation lines and planes) is the separation threefold. Hence it is obvious that only separation threefolds *could* be Euclidean. In other kinds of threefolds analogies to Euclid I, 47, hold, but they are only analogies.

At length coordinates can be introduced. We take three mutually normal separation lines in a separation threefold as our x , y , z axes. And we take an inertia line normal to this threefold as a t axis. But we must notice that, since congruence means something different for inertia and for separation segments, we cannot use the same unit for distances along the t axis as for those along the other three. What we do is to choose such a unit for our inertia coordinates that the conjugate to it (which is necessarily a separation segment) is c times the unit separation segment, where c is a constant. The constant will then be the numerical measure of the velocity of light.

(3) What precisely has Mr. Robb accomplished and what is the philosophic importance of his work? It seems to me that his results and their importance may be expressed somewhat as follows: Modern science has inherited from the founders of mechanics in the XVIIth century and from the Greek founders of geometry a certain general scheme of dealing with the physical world. This scheme treats the ultimate elements of physics as particles occupying various places in a three-dimensional space at various moments in a one-dimensional time. The time and the space are separate systems and neither is given to us in experience. This is true in three senses: (1) We never directly perceive a moment or a point. (2) We never directly perceive even aggregates of moments or points. (3) It is true that we perceive extended objects in spatial relations and are aware of the duration and succession of certain events. But our special way of interpreting these facts, *viz.* our view that the events take place at a certain moment in a single time and at certain points in a common space, is a construction and not something that can be analysed out of our sense-data. We do not perceive it, nor can it in any useful and natural sense be called a part of what we perceive.

This general scheme worked excellently in practice owing to a happy choice of spatial coordinates and to the fact that people were mainly concerned with velocities small in comparison with that of light. Accordingly its peculiar nature and its presuppositions were not much noticed until certain electromagnetic experiments were found to lead to very paradoxical conclusions. Then people were led to see much more clearly that all measurements

of distance make certain assumptions about time, and all measurement of time which refer to different places involve assumptions about spatial measurement. It thus becomes clear that we shall keep much closer to the empirical facts if we no longer start by assuming two different kinds of entities (instants with their temporal relations and points with their spatial relations). We shall do better if we start with elements of a single kind which come nearer to what we can actually observe, and by subjecting them to suitable postulates construct both the ordinary space and the ordinary time out of them. Construction here means nothing specially human. It means (a) that knowing approximately the results that are true we lay down the postulates that we think will give them, and (b) that the space and time of ordinary physics appear (with such modifications as experience demands) as special cases in the general scheme.

The great merit of Mr. Robb's work is that he has actually provided us with an alternative construction of this kind and shown us that it will fit all the facts at present recognised. And the philosophic importance of all such attempts is that, like the study of non-Euclidean geometry, they free the mind from ingrained prejudice and enable it to see that what appears a necessity of thought is often only one of a number of alternative ways of dealing with a single problem.

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What do we mean by Education? By J. WELTON. London: Macmillan, 1915. Pp. xii, 257.

AN increasing number of teachers and educational administrators are taking a keen interest in the theoretical aspects of their work, and to such readers Prof. Welton's book should make a strong appeal. It will also serve as a useful guide for readers who are not actively engaged in education, but who desire to keep in touch with the wider movements of educational thought and practice. For both classes of students the work is valuable mainly because it is the fruit of a serious effort to view educational problems in the light of a more or less definite conception of human life. "Theory of education," Prof. Welton tells us, "cannot be separated without disaster from theory of life," and he puts his doctrine into practice with the help of much hard thinking and a long experience of educational work. His criticism of one-sided and exaggerated views is particularly valuable. If the enthusiastic advocates of educational panaceas could be induced to digest his book, the outlook for school reform would become appreciably brighter.

I wish to emphasise these merits of the book before us, because they are by no means neutralised by certain weaknesses which